American Journal of Modern Physics
2023; 12(3): 47-53
http://www.sciencepublishinggroup.com/j/ajmp
doi: 10.11648/j.ajmp.20231203.12
ISSN: 2326-8867 (Print); ISSN: 2326-8891 (Online)

Annular Axisymmetric Stagnation Flow of a Casson Fluid Through Porous Media in a Moving Cylinder

Muhammad Umar Farooq*, Abdul Rehman, Naveed Sheikh

Department of Mathematics, University of Balochistan, Quetta, Pakistan

Email address:
Umarfarooq71965@gmail.com (Muhammad Umar Farooq), abdul_maths@yahoo.com (Abdul Rehman),
naveed.maths@gmail.com (Naveed Sheikh)

*Corresponding author

To cite this article:
Muhammad Umar Farooq, Abdul Rehman, Naveed Sheikh. (2023). Annular Axisymmetric Stagnation Flow of a Casson Fluid Through Porous Media in a Moving Cylinder. *American Journal of Modern Physics*, *12*(3), 47-53. https://doi.org/10.11648/j.ajmp.20231203.12

Received: November 14, 2023; **Accepted**: December 4, 2023; **Published**: December 14, 2023

Abstract: The present investigation deals with the study of annular axisymmetric stagnation flow and heat transfer of a Casson fluid through porous medium in a moving cylinder. The inner cylinder is rotating with a fixed angular velocity and is also translating along axial axis with constant axial velocity while the outer cylinder is assumed fixed. Fluid is injected from the top surface of the fixed outer cylinder towards the translating and rotating inner cylinder. The motion of the Casson fluid is assumed under the influence of some porous media. The governing nonlinear partial differential equations of conservation of mass, conservation of li near momentum and heat transfer are obtained and then simplified with the help of a set of suitable similarity transformations which reduces the original set of partial differential equations into a new simplified set of ordinary differential equations. The resulting system of ordinary differential equations is then solved numerically with the aid of fifth order Runge-Kutta Fehlberg method. A comparison of special cases of the present numerical solution with the already available work is also included through tables. The behavior of important involved physical non-dimensional parameters like Prandtl numbers, porosity parameter and Reynolds numbers is also presented at the end.

Keywords: Axisymmetric Stagnation Flow, Casson Fluid, Moving Cylinder, Porous Medium

1. Introduction

The study of stagnation point flow was initiated by Hiemenz [1]. Since then many researchers have contributed to the study of stagnation point flow for viscous and non-Newtonian fluids for different flow geometries. Weidman and Putkaradze [2] have classified stagnation flows as viscous or inviscid, steady or unsteady, two or three dimensional, symmetric or asymmetric, normal or oblique, homogenous or inhomogeneous and forward or reverse. In certain situations, fluid is been stagnated by the surface of the object, while in some cases a free stagnation point or line exists in the interior of the fluid domain. The problem of mixed convection stagnation point flow of a non-Newtonian micropolar nanofluid flowing over a vertical slender cylinder is studied by Rehman and Nadeem [3]. In another work, Robert et al [4] have analyzed the hydromagnetic stagnation point flow of a second grade fluid over a stretching sheet. Chiam [5] has examined the variable conductivity heat transfer in a stagnation flow towards a stretching sheet. In a very recent work, Rehman et al [6] have presented the solution for the problem of boundary layer stagnation flow of a third grade fluid that is flowing over an exponentially stretching surface. The non-orthogonal stagnation point flow towards a stretching vertical plat was examined by Yian et al [7]. The results obtained in [7] indicate that when the stretching velocity of the surface is greater than the free stream stagnation velocity the boundary layer gains an inverted structure. Few other interesting works concerning the stagnation flows are referred in [8-22].

Recently, Nadeem et al [23] have discussed the axisymmetric stagnation flow of a micropolar fluid containing nanoparticles and is flowing through the annular region between the concentric cylinders. The present work provides an investigation for the flow of a non-Newtonian Casson fluid flowing through porous medium. The fluid is assumed to be flowing through the annular region between the two concentric cylinders. The infinite inner cylinder is rotating as well as translating along the axial direction while the finite outer cylinder is kept fixed. The solution of such finite domain stagnation flow problem is very rare. For solution of the problem, first the governing equations are nondimensionalized using a similarity transformation; the obtained system of ordinary differential equations is solved using the numerical scheme the Fehlberg method. The behavior of velocity and temperature profiles is discussed in detail for the parameters involved at the end.

2. Formulation

Consider an incompressible flow of a Casson fluid between two cylinders flowing through porous medium. The flow is assumed to be axisymmetric about z -axis. The inner cylinder is of radius R rotating with an angular velocity Ω and moving with velocity W in the axial z -direction. The inner cylinder is enclosed by an outer cylinder of radius $b_0 R$. The fluid is injected radially with velocity U from the outer cylinder towards the inner cylinder. The governing equations of motion and heat transfer are [24-30].

$$rw_z + (ru)_r = 0, \tag{1}$$

$$uu_r + wu_z - \frac{v^2}{r} = -\frac{1}{\rho} p_r + \nu(1+\frac{1}{\beta})(u_{rr} + \frac{1}{r}u_r + u_{zz} - \frac{u}{r^2}) - \frac{\nu\varphi_p}{k_0}u, \tag{2}$$

$$uv_r + wv_z + \frac{uv}{r} = \nu(1+\frac{1}{\beta})(v_{rr} + \frac{1}{r}v_r + v_{zz} - \frac{v}{r^2}) - \frac{\nu\varphi_p}{k_0}v, \tag{3}$$

$$uw_r + ww_z = -\frac{1}{\rho} p_z + \nu(1+\frac{1}{\beta})(w_{rr} + \frac{1}{r}w_r + w_{zz}) - \frac{\nu\varphi_p}{k_0}w, \tag{4}$$

$$uT_r + wT_z = \alpha(T_{rr} + \frac{1}{r}T_r + T_{zz}), \tag{5}$$

where (u,v,w) are the velocity components along (r,θ,z) axes, μ is the viscosity, ρ is the density, ν is the kinematic viscosity, p is pressure, β is the Casson fluid parameter, φ_p is porosity of porous space, k_0 is permeability of porous space, α is the thermal diffusivity and T is temperature. The corresponding boundary conditions are

$$u(R,z) = 0, \quad v(R,z) = \Omega a, \quad w(R,z) = W, \quad T(R,z) = T_1, \tag{6}$$

$$u(b_0R,z) = -U, \quad v(b_0R,z) = 0, \quad w(b_0R,z) = 0, \quad T(b_0R,z) = T_2, \tag{7}$$

where T_1 is the fluid temperature at the surface of the inner cylinder while T_2 is the fluid temperature at the surface of the outer cylinder.

3. Solution of the Problem

The system of nonlinear partial differential equations $(1-5)$ subject to the boundary conditions $(6-7)$ are simplified using the similarity transformations [16]

$$u = -\frac{Uf(\eta)}{\sqrt{\eta}}, \quad v = \Omega a h(\eta), \quad w = 2Uf'(\eta)\xi + Wg(\eta), \tag{8}$$

$$\theta = \frac{T-T_1}{T_b - T_1}, \quad \eta = \frac{r^2}{R^2}, \quad \xi = \frac{z}{R}. \tag{9}$$

With the help of above transformation, $Eq.(1)$ is identically satisfied and $Eqs.(2)$ to (5) take the following form

$$(1+\frac{1}{\beta})(\eta f^{(IV)} + 2f''') + \mathrm{Re}(ff''' - f'f'') - kpf'' = 0, \tag{10}$$

$$(1+\frac{1}{\beta})(\eta g'' + g') + \mathrm{Re}\left(fg' - f'g\right) - kpg = 0, \tag{11}$$

$$(1+\frac{1}{\beta})(\eta h'' + h' - \frac{h}{4\eta}) + \mathrm{Re}\left(2fh' + \frac{fh}{\eta}\right) - kph = 0, \tag{12}$$

$$\eta\theta'' + \theta' + \mathrm{Pr}\,\mathrm{Re}\, f\theta' = 0, \tag{13}$$

in which $\mathrm{Re} = UR/2\nu$ is the Reynolds number, $kp = R^2\varphi_p / 4k_0$ is the porosity parameter and $\mathrm{Pr} = \nu/\alpha$ is the Prandtl number. The boundary conditions in nondimensional form are

$$f(1) = 0, \; f'(1) = 0, \; g(1) = 1, \; h(1) = 1, \; \theta(1) = 0, \tag{14}$$

$$f(b) = \sqrt{b}, \; f'(b) = 0, \; g(b) = 0, \; h(b) = 0, \; \theta(b) = 1. \tag{15}$$

Where $\sqrt{b} = b_0$. For numerical solution of the problem, we consider $F_1 = f'$, $F_2 = F_1'$, $F_3 = F_2'$, $G = g'$, $H = h'$ and $\varphi = \theta'$ then the resulting system is

$$(1+\frac{1}{\beta})(\eta F_3' + 2F_3) + \mathrm{Re}(fF_3 - F_1F_2) - kpF_2 = 0, \quad (16)$$

$$(1+\frac{1}{\beta})(\eta G' + G) + \mathrm{Re}\left(fG - F_1 g\right) - kpg = 0, \quad (17)$$

$$(1+\frac{1}{\beta})(\eta H' + H - \frac{h}{4\eta}) + \mathrm{Re}\left(2fH + \frac{fh}{\eta}\right) - kph = 0, \quad (18)$$

$$\eta\varphi' + \varphi + \mathrm{Pr}\,\mathrm{Re}\, f\varphi = 0, \quad (19)$$

The above system of first order odes is solved numerically with the help of fifth order Fehlberg method and the obtained results are discussed in the next section.

4. Results and Discussion

The numerical solution of the problem of steady, incompressible, non-Newtonian Casson fluid flowing through the annular region between the concentric cylinders is obtained through the fifth order Fehlberg method. The outer cylinder is assumed to be fixed while the inner cylinder is translating with a constant axial velocity and is also rotating axisymmetrically about the axial direction through some porous medium. The behavior of non-dimensional velocity profiles for the involved parameters is plotted in figures 1 and 2 [31-35]. In these figures f is associated with the fluid flow in radial direction, g is concerned with the fluid flow in axial direction while h represents the rotation effects in the fluid flow caused due to the rotation of inner cylinder. Figures (1-3) show the behavior of radial velocity with respect to the Reynolds numbers Re, Casson fluid parameter β and the porosity parameter kp respectively. From figure 1 it is noted that with the increase in Reynolds numbers Re the non-dimensional radial velocity increases. This happen because higher radial velocity is concerned with a higher injunction velocity (and so a higher Reynolds number) from outer cylinder towards the inner cylinder. Figure 2 shows that with increase in the Casson fluid parameter β increases the radial velocity f. figure 3 indicates that by increasing the porosity parameter kp the radial velocity decreases because higher kp corresponds to porous medium with lesser permeability and thus fluid flow encounters more resistance. The influence of Reynolds numbers Re over the velocity gradient f' is sketched in figure 4 that indicates that near the surface of inner cylinder velocity gradient increases with the increase in Re while near the surface of outer cylinder, increase in Re decreases the velocity gradient. Figures 5 and 6 give the impact of Casson fluid parameter β and the porosity parameter kp over the velocity gradient f'. From these graphs it is observed that near the surface of inner cylinder increase in β increases f' while increase in kp decreases f', whereas near the surface of outer cylinder, increase in β decreases f' while increase in kp increases f'. figures 7-9 show the behavior of axial velocity profile g for Reynolds numbers Re, Casson fluid parameter β and the porosity parameter kp. From these plots it is noticed that the axial velocity g decreases with increase in all the parameters Re, β and kp. figures 10-12 shows the pattern adopted by the non-dimensional rotation velocity h for the involved parameters Re, β and kp respectively. From these figures it is evident that with increase in all the three parameters the angular velocity profile h decreases. The variation in temperature flow θ for Prandtl numbers Pr and Reynolds numbers Re is presented in figures 13 and 14 From these sketches it is noted that with increase in both Pr and Re the non-dimensional temperature profile increases [36-42].

To validate the accuracy of the current numerical results the present Fehlberg solutions are compared as a special case with the existing work of Hong and Wang [16] in table 1 The boundary derivatives obtained for the velocity profiles are in excellent agreement. Table 2 predicts the behavior of boundary derivatives for the velocity profiles f and g computed at the surface of the inner cylinder that proportional to the skinfriction at the surface of the inner cylinder computed for different values of the Casson fluid parameter β and the porosity parameter kp. From table 2 it is evident that increasing β and kp also increases the skinfriction coefficient. Table 3 contains the values for the boundary derivative of the angular velocity profile that are computed for different Reynolds numbers and Prandtl numbers and that corresponds to the azimuthal shear stress at the surface of the inner cylinder. From table 3 it is observed that increase in both Re and Pr increases the azimuthal shear stress. Table 4 shows the numerical values of the boundary derivative of the temperature flow function θ that predicts the behavior of Nusselt numbers Nu against Prandtl numbers Pr and Reynolds numbers Re. From table 4 it is observed that the Nusselt numbers increases with the increase in both Pr and Re [43-45].

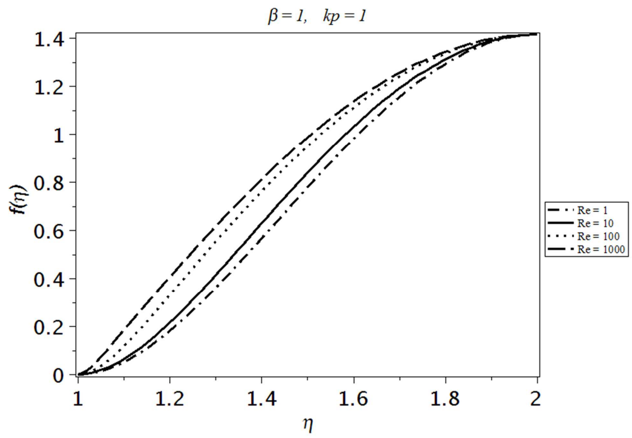

Figure 1. *Influence of Re over f.*

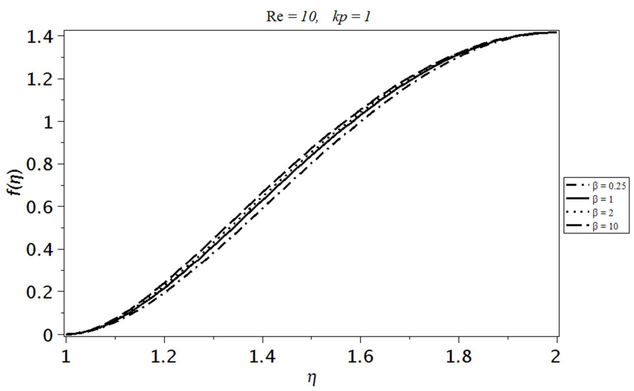

Figure 2. *Influence of* β *over f.*

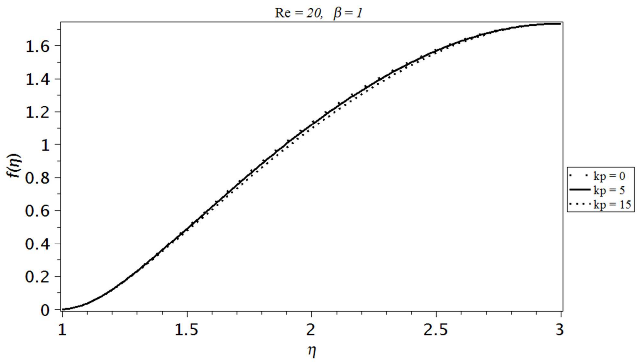

Figure 3. *Influence of* k_p *over f.*

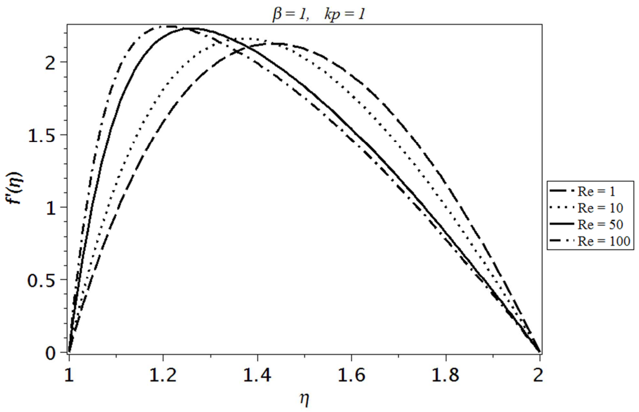

Figure 4. *Influence of Re over f'.*

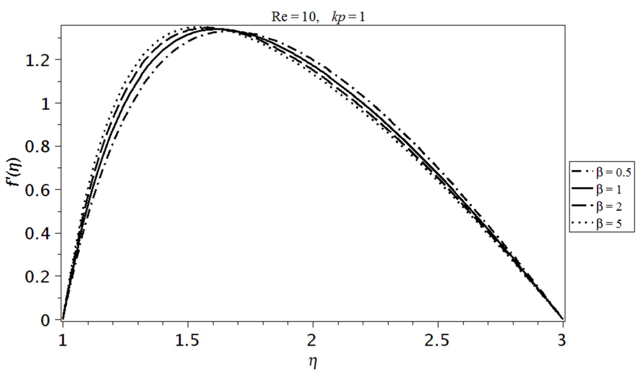

Figure 5. *Influence of* β *over f'.*

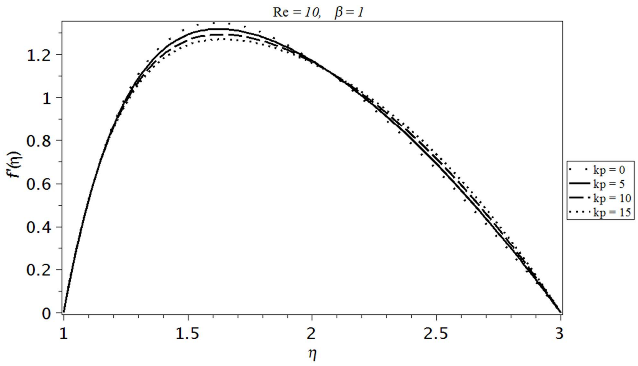

Figure 6. *Influence of* k_p *over f'.*

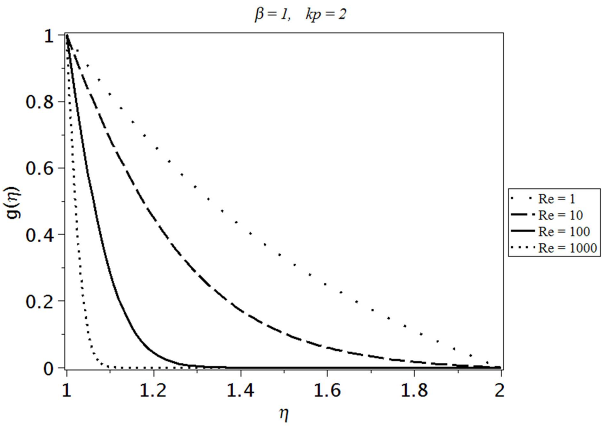

Figure 7. *Influence of Re over g.*

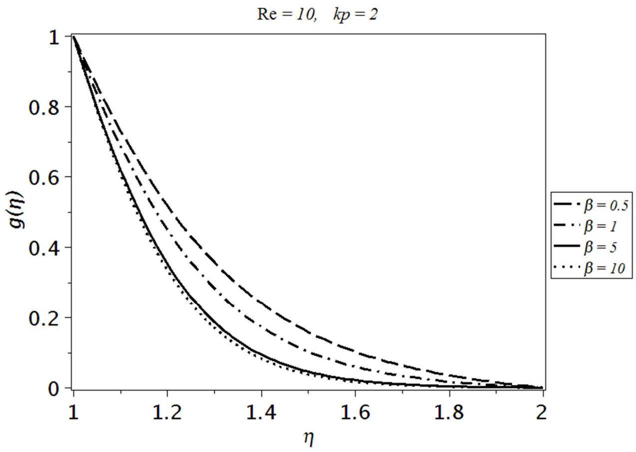

Figure 8. *Influence of* β *over g.*

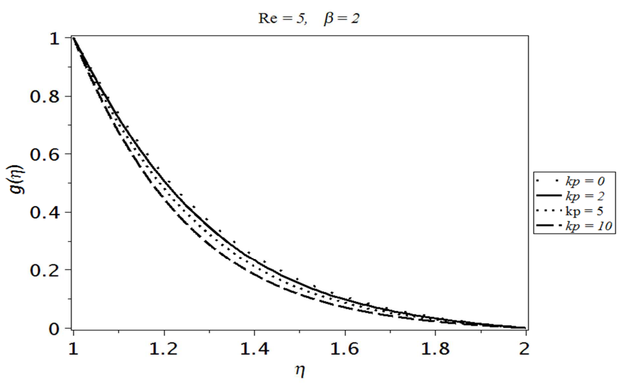

Figure 9. *Influence of* k_p *over g.*

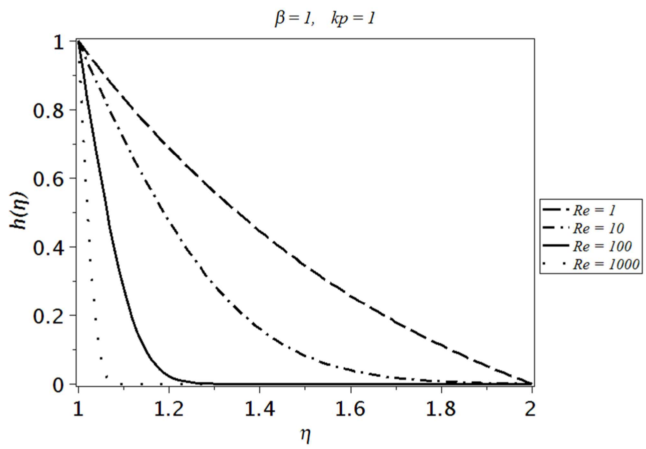

Figure 10. *Influence of Re over h.*

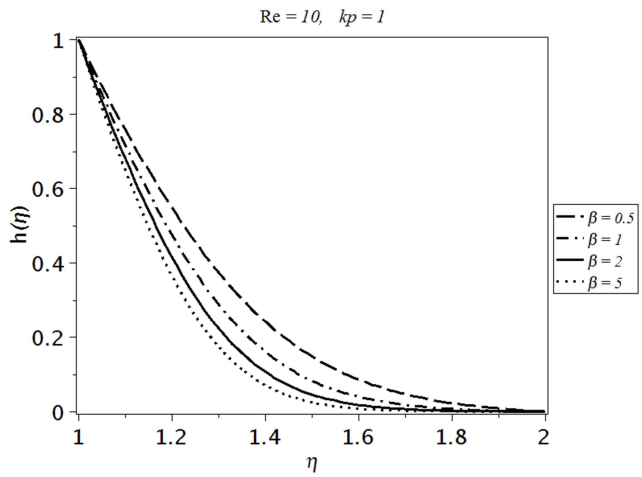

Figure 11. *Influence of β over h.*

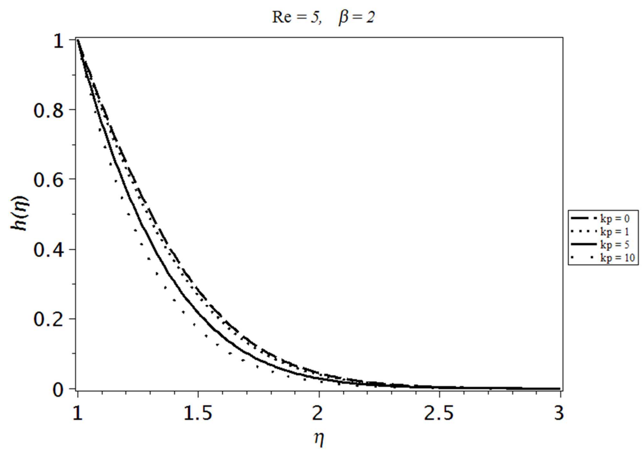

Figure 12. *Influence of k_p over h.*

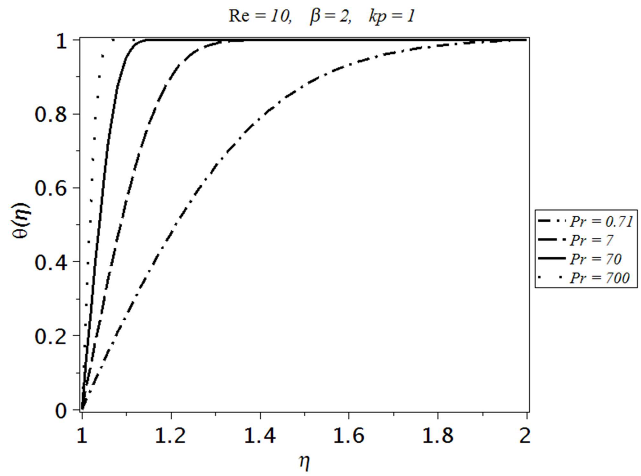

Figure 13. *Influence of Pr over θ.*

Table 1. *Comparison of the boundary derivatives of the present results with the work in [16] when β=20000 and k_p=0.*

β/kp	**f´´(1)**					**-g´(1)**				
	0.2	**0.5**	**1**	**2**	**5**	**0.2**	**0.5**	**1**	**2**	**5**
0.5	13.366	13.374	13.386	13.409	13.481	2.805	2.821	2.847	2.899	3.051
1	14.499	14.509	14.523	14.552	14.639	3.288	3.308	3.341	3.407	3.598
5	16.583	16.593	16.608	16.640	16.739	4.071	4.097	4.141	4.226	4.476
50	17.431	17.434	17.455	17.487	17.586	4.367	4.395	4.442	4.535	4.807
500	17.530	17.539	17.554	17.585	17.685	4.401	4.429	4.477	4.571	4.845

Table 2. *Behavior of boundary derivatives for linear velocity profiles computed for different β and k_p when Re=10.*

Re	**f´´(1)**		**-f´´´(1)**		**-g´(1)**		**-h´(1)**	
	Present	**[16]**	**Present**	**[16]**	**Present**	**[16]**	**Present**	**[16]**
0.1	11.001	11.0010	36.1443	36.1443	1.4963	1.4963	10.5151	10.5151
1	11.677	11.6772	41.0795	41.0797	1.9309	1.9309	10.6511	10.6511
10	17.5345	17.5348	93.565	93.5670	4.3856	4.3856	12.0407	12.0407
100	44.449	44.4492	790.731	590.738	12.6450	12.6450	24.8226	24.8226

Table 3. *Behavior of boundary derivatives for angular velocity profiles computed for different Pr and Re when β=5.*

Re/kp	**-h´(1)**				
	0.2	**0.5**	**1**	**2**	**5**
0.5	1.6705	1.7335	1.8358	2.0309	2.5534
1	1.7987	1.8584	1.9555	2.1412	2.6422
5	2.7703	2.8095	2.8740	3.0003	3.3588
10	3.6861	3.7142	3.7608	3.8529	4.1216
100	10.5678	10.5766	10.5912	10.6203	10.7076

Table 4. *Behavior of boundary derivatives for temperature profile computed for different pr and re when β=5, kp=2.*

Re/Pr	(1)				
	0.7	**7**	**20**	**70**	**700**
0.5	1.5063	2.0426	2.8658	4.4448	9.7080
1	1.5707	2.5416	3.6821	5.6738	12.3513
5	2.0910	4.6744	6.7300	10.3204	22.4300
10	2.6897	6.2203	8.9540	13.7367	29.8783
100	7.6624	17.8594	25.7870	39.6852	

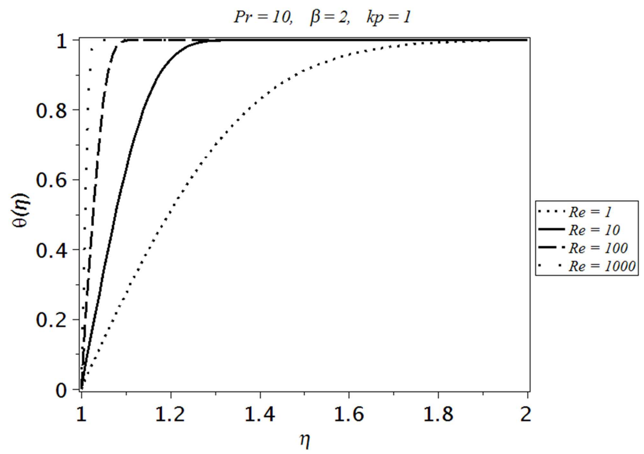

Figure 14. *Influence of Re over θ.*

5. Conclusion

In the present paper the authors have analyzed the problem of Casson fluid flow through the annular region formed between to concentric cylinders. Main findings of the study are as under

1. With increase in the porosity parameter the Reynolds number the angular velocity decreases
2. With increase in the Prandtl number the temperature profile the thermal layer decreases.

Conflicts of Interest

The authors declare no conflicts of interest.

References

[1] K. Hiemenz, Die Grenzschicht in Einem in Dem Gleichformingen Flussigkeitsstrom Eingetauchten Gerade Kreiszylinder, Dingler Polytech J 326 (1911) 321-410.

[2] P. D. Weidman, V. Putkaradze, Axisymmetric stagnation flow obliquely impinging on a circular cylinder, Eur. J. Mech. B-fluids, 22 (2) (2003) 123-131.

[3] Abdul Rehman, S. Nadeem, Mixed convection heat transfer in micropolar nanofluid over a vertical slender cylinder, Chin. Phys. Lett. 29 (12) (2012) 124701.

[4] Robert A. Van Gorder, K. Vajravelu, Hydromagnetic stagnation point flow of a second grade fluid over a stretching sheet, Mech. Res. Commun. 37 (2010) 113-118.

[5] T. C. Chiam, Heat transfer with variable conductivity in a stagnation-point flow towards a stretching sheet, Int. Comm. Heat Mass Tran. 23 (2) (0996) 239-248.

[6] Abdul Rehman, S. Nadeem, M. Y. Malik, Boundary layer stagnation-point flow of third grade fluid over an exponentially stretching sheet, Brazilian J. Chem. Eng.

[7] L. Y. Yian, N. Amin, D. Filip, I. Pop, Mixed convection flow near a non-orthogonal stagnation point towards a stretching vertical plate. Int J Heat Mass Tran. 50 (2007) 4855-63.

[8] M. Suali, N. M. A. Nik Long, A. Ishak, Unsteady stagnation point flow and heat transfer over a stretching/shrinking sheet with prescribed surface heat flux, App. Math. Comp. Intel., 1 (2012) 1-11.

[9] Abdul Rehman, R. Bazai, S. Achakzai, S. Iqbal, M. Naseer, Boundary Layer Flow and Heat Transfer of Micropolar Fluid over a Vertical Exponentially Stretched Cylinder, App Comp Math, 4 (6) (2015) 424-430.

[10] Zhu, J., Zheng, L. C., and Zhang, Z. G. Effects of slip condition on MHD stagnation-point flow over a power-law stretching sheet. Applied Mathematics and Mechanics (English Edition), 31 (4), 439–448 (2010) DOI 10.1007/s104830100404z.

[11] H. A. Attia, On the effectiveness of porosity on stagnation point flow towards a stretching surface with heat generation, Comp. Mater. Sci. 38 (2007) 741-745.

[12] S. Nadeem, Abdul Rehman, C. Lee, J. Lee, Boundary layer flow of second grade fluid in a cylinder with heat transfer, Math. Prob. Eng. 2012, Article ID 640289, 13 pages, doi: 10.1155/2012/640289.

[13] S Nadeem, Abdul Rehman, Mohamed E Ali, The boundary layer flow and heat transfer of a nanofluid over a vertical, slender cylinder, DOI: 10.1177/1740349912453806.

[14] Zhu, J., Zheng, L. C., and Zhang, X. X. Analytical solution to stagnation-point flow and heat transfer over a stretching sheet based on homotopy analysis. Applied Mathematics and Mechanics (English Edition), 30(4), 463–474 (2009) DOI 10.1007/s1048300904072.

[15] Abdul Rehmana, S. Achakzai, S. Nadeem, S. Iqbal, Stagnation point flow of Eyring Powell fluid in a vertical cylinder with heat transfer, Journal of Power Technologies 96 (1) (2016) 57–62.

[16] L. Hong, C. Y. Wang, Annular axisymmetric stagnation flow on a moving cylinder, Int. J. Engag. Sci. 47 (2009) 141-152.

[17] S. Nadeem, Abdul Rehman, Axisymmetric stagnation flow of a nanofluid in a moving cylinder, Comp. Math. Mod. 24 (2) (2013) 293-306.

[18] M. Y. Malik, M. Naseer, S. Nadeem, Abdul Rehman, The boundary layer flow of Casson nanofluid over a vertical exponentially stretching cylinder, Appl. NanoSci. DOI: 10.1007/s13204-012-0267-0.

[19] Abdul Rehman, S. Nadeem, S. Iqbal, M. Y. Malik, M. Naseer, Nanoparticle effect over the boundary layer flow over an exponentially stretching cylinder, J. NanoEngineering and NanoSystems (2014) 1-6.

[20] M. Y. Malik, M. Naseer, S. Nadeem, Abdul Rehman, The boundary layer flow of hyperbolic tangent fluid over a vertical exponentially stretching cylinder, Alexandria Eng. J., 53 (2014) 747-750.

[21] M. Y. Malik, M. Naseer, Abdul Rehman, Numerical study of convective heat transfer on the Power Law fluid over a vertical exponentially stretching cylinder, App Comp Math, 4 (5), (2015) 346-350.

[22] Abdul Rehman, S. Nadeem, M. Y. Malik, Stagnation flow of couple stress nanofluid over an exponentially stretching sheet through a porous medium, J. Power Tech. 93 (2) (2013) 122-132.

[23] S. Nadeem, Abdul Rehman, K. Vajravelu, J. Lee, C. Lee, Axisymmetric stagnation flow of a micropolar nanofluid in a moving cylinder, Math. Prob. Eng. 2012, Article ID 378259, 18 pages, doi: 10.1155/2012/378259.

[24] Abdul Rehman, S. Nadeem, Heat transfer analysis of the boundary layer flow over a vertical exponentially stretching cylinder, Global J. Sci. Fron. Res. 13 (11) (2013) 73-85.

[25] Abdul Rehman, G. Farooq, I. Ahmed, M. Naseer, M. Zulfiqar, Boundary Layer Stagnation-Point Flow of Second Grade Fluid over an Exponentially Stretching Sheet, American J App Math Stat, 3 (6) (2015) 211-219.

[26] Abdul Rehman, Saleem Iqbal, Syed Mohsin Raza, Axisymmetric Stagnation Flow of a Micropolar Fluid in a Moving Cylinder: An Analytical Solution, Fluid Mechanics, 2 (1) (2016) 1-7.

[27] Muhammad Umar Farooq, Abdul Rehman, Naveed Sheikh, Manzoor Iqbal, Axisymmetric Flow through a Cylinder with Porous Medium, International Journal of Systems Science and Applied Mathematics, 2020; 5 (3): 32-35.

[28] Naheeda Iftikhar, Abdul Rehman, Peristaltic flow of an Eyring Prandtl fluid in a diverging tube with heat and mass transfer, International Journal of Heat and Mass Transfer 111 (2017) 667–676.

[29] Abdul Rehman, Naveed Sheikh, Boundary Layer Stagnation-Point Flow of Micropolar Fluid over an Exponentially Stretching Sheet, International Journal of Fluid Mechanics & Thermal Sciences, 2017; 3 (3): 25-31.

[30] Haroon Rasheed, Abdul Rehman, Naveed Sheikh, Saleem Iqbal, MHD Boundary Layer Flow of Nanofluid over a Continuously Moving Stretching Surface, Applied and Computational Mathematics, 2017; 6 (6): 265-270.

[31] Naheeda Iftikhar, Abdul Rehman, Hina Sadaf, Muhammad Najam Khan, Impact of wall properties on the peristaltic flow of Cu-water nano fluid in a non-uniform inclined tube, International Journal of Heat and Mass Transfer 125 (2018) 772–779.

[32] Amina Panezai, Abdul Rehman, Naveed Sheikh, Saleem Iqbal, Israr Ahmed, Manzoor Iqbal, Muhammad Zulfiqar, Mixed Convective Magnetohydrodynamic Heat Transfer Flow of Williamson Fluid Over a Porous Wedge, American Journal of Mathematical and Computer Modelling, 4 (3) (2019) 66-73.

[33] N. Ambreen, A. Rehman, N. Sheikh, S. Iqbal, M. Zulfqar, Boundary-Layer Flow and Heat Transfer over a Rotating Porous Disk in a Non-Newtonian Williamson Nanofluid, Indian Journal of Science and Technology, 12 (38) (2019) 1-8.

[34] Zahida Khan, Abdul Rehman, Naveed Sheikh, Saleem Iqbal, Ejaz Shah, Boundary Layer Flow of a Nanofluid through a Permeable Medium Due to Porous Plate, American Journal of Mathematical and Computer Modelling, 2020; 5 (4): 97-101.

[35] Abdul Rehman, S. Nadeem, Mixed convection heat transfer in micropolar nanofluid over a vertical slender cylinder, Chin. Phy. Lett. 29 (12) (2012) 124701-5.

[36] S. Nadeem, Abdul Rehman, Mohamed Ali, The boundary layer flow and heat transfer of a nanofluid over a vertical slender cylinder, J. NanoEngineering and NanoSystems (2012) 1-9.

[37] Naheeda Iftikhar, Abdul Rehman and Muhammad Najam Khan, Features of Convective heat transfer on MHD peristaltic movement of Williamson fluid with the presence of Joule heating, IOP Conf. Series: Materials Science and Engineering 414 (2018) 012010.

[38] Najeeb Alam Khan, Umair Bin Saeed, Faqiha Sultan, Saif Ullah, and Abdul Rehman, Study of velocity and temperature distributions in boundary layer flow of fourth grade fluid over an exponential stretching sheet, AIP ADVANCES 8, 025011 (2018).

[39] Naheeda Iftikhar, Abdul Rehman, Hina Sadaf, Theoretical investigation for convective heat transfer on Cu/water nanofluid and (SiO2-copper)/water hybrid nanofluid with MHD and nanoparticle shape effects comprising relaxation and contraction phenomenon, International Communications in Heat and Mass Transfer, 120 (2021) 105012.

[40] Naheeda Iftikhar, Abdul Rehman, Hina Sadaf, and Saleem Iqbal, Study of Al2O3/copper–water nanoparticle shape, slip effects, and heat transfer on steady physiological delivery of MHD hybrid nanofluid, Canadian Journal of Physics, 97 (12) (2019) 1239-1252.

[41] Farah Deba, Abdul Rehman, Zahida Khan, Saleem Iqbal, Naveed Sheikh, Flow of a Williamson Fluid over a Stretching Sheet Containing Nanoparticles, International Journal of Systems Engineering, 5 (1) (2021) 13-17.

[42] Abdur Rab, Abdul Rehman, Zahida Khan, Non-Newtonian Fluid Flow through Cylindrical Surface in Presences of Porous Material, European Academic Research, IX (10) (2022) 6428-6438.

[43] Naheeda Iftikhar, Abdul Rehman, Hina Sadaf, Inspection of physiological flow in the presence of nanoparticles with MHD and slip effects, Journal of Thermal Analysis and Calorimetry, 147 (2022) 987–997.

[44] Ghulam Shah, Abdul Rehman, Naveed Sheikh, Heat Transfer Analysis over the Boundary Layer Stagnation-Point Flow of Couple Stress Fluid over an Exponentially Stretching Sheet, American Journal of Applied Mathematics, 10 (3) (2022) 100-105.

[45] M. Rafiq, Abdul Rehman, N. Sheikh, M. Saleem, M. Umar Farooq, Numerical Study of the Boundary Layer Flow Problem over a Flat Plate by Finite Difference Method, Applied Engineering, 7 (2) (2023) 27-36.